AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TARGET - JEE- 02 TEST - 03
SOLUTION Date 21-07-2019

[PHYSICS]

1. $\text{Average speed} = \dfrac{\text{total distance covered}}{\text{total time taken}}$

$$v_{av.} = \frac{\frac{x}{2}+\frac{x}{2}}{\frac{x/2}{40}+\frac{x/2}{60}} = \frac{x}{\left(\frac{x}{80}+\frac{x}{120}\right)}$$

$$= \frac{80\times 120}{(120+80)} = 48\ \text{km/h}$$

2. $200 = u \times 2 - (1/2)\, a(2)^2$ or $u - a = 100$ ....(i)

$200 + 220 = u(2 + 4) - (1/2)\,(2 + 4)^2 a$

or $u - 3a = 70$ .....(ii)

Solving eqns. (i) and (ii), we get; $a = 15\ \text{cm/s}^2$ and $u = 115$ cm/s.

Further, $v = u - at = 115 - 15 \times 7 = 10$ cm/sec.

3. When a body slides on an inclined plane, component of weight along the plane produces an acceleration

$$a = \frac{mg\sin\theta}{m} = g\sin\theta = \text{constt.}$$

If s be the length of the inclined plane, then

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta \times t^2$$

$$\therefore \quad \frac{s'}{s} = \frac{t'^2}{t^2} \text{ or } \frac{s}{s'} = \frac{t^2}{t'^2}$$

Given t = 4 sec and $s' = \dfrac{s}{4}$

$$\therefore \quad t' = t\sqrt{\frac{s'}{s}} = 4\sqrt{\frac{s}{4s}} = \frac{4}{2} = 2\,\text{sec}$$

4. Given that; $a = 3t + 4$ or $\dfrac{dv}{dt} = 3t + 4$

$$\therefore \quad \int_0^v dv = \int_0^t (3t+4)dt \text{ or } v = \frac{3}{2}t^2 + 4t$$

$$v = \frac{3}{2}(2)^2 + 4(2) = 14\ ms^{-1}$$

5. **For first body** :

$$\frac{1}{2}gt^2 = 176.4 \quad \text{or} \quad t = \sqrt{\frac{176.4 \times 2}{10}}$$

or $t = 5.9$ s

For second body : $t = 3.9$ s

$$u(3.9) + \frac{1}{2}g(3.9)^2 = 176.4$$

$$3.9u + \frac{10}{2}(3.9)^2 = 176.4$$

or $u = 24.5$ m/s

6. The resultant velocity of the boat and river is 1.0 km/0.25 h = 4 km/h.

Velocity of the rive $= \sqrt{5^2 - 4^2} = 3\ km/h$

7. Let he be the height of the tower.
Using $v^2 - u^2 = 2as$, we get;
Here, $u = u$, $a = -g$, $s = -h$ and $v = -3u$ (upward direction + ve)
$\therefore$ $9u^2 - u^2 = 2gh$ or $h = 4u^2/g$

8. $t = \sqrt{\frac{2h}{g}}$

$$s = 10 \times \frac{t}{2} - \frac{1}{2}g \times \frac{t^2}{4} = 5\sqrt{\frac{2h}{g}} - \frac{g}{8}\frac{2h}{g}$$

$v^2 - u^2 = 2gh$ or $100 = 2gh$ or $10 = \sqrt{2gh}$

$$s = \sqrt{\frac{2gh \times 2h}{4 \times g}} - \frac{h}{4} = h - \frac{h}{4} = \frac{3h}{4}$$

9. $t = \frac{1}{u+v} = \frac{1}{\frac{l}{t_1} + \frac{l}{t_2}}$

or $\frac{1}{t} + \frac{1}{t_1} + \frac{1}{t_2}$ or $t = \frac{t_1 t_2}{(t_1 + t_2)}$

10. **For first body** :
$v^2 = u^2 + 2gh$ or $(3)^2 = 0 + 2 \times 9.8 \times h$

or $h = \frac{(3)^2}{2 \times 9.8} = 0.46$ m

For second body :
$v^2 = (4)^2 + 2 \times 9.8 \times 0.46$

$\therefore$ $v = \sqrt{(4)^2 + (2 \times 9.8 \times 0.46)} = 5$ m/s

11. Given y = 0

Distance travelled in 10 s,

$$S_1 = \frac{1}{2}a \times 10^2 = 50a$$

Distance travelled in 20 s,

$$S_2 = \frac{1}{2}a \times 20^2 = 200a$$

$\therefore$ $S_2 = 4S_1$

12. During the first 5 seconds of the motion, the acceleration is – ve and during the next 5 seconds it becomes positive. (Example : a stone thrown upwards, coming to momentary rest at the highest point). The distance covered remains same during the two intervals of time.

13. Gain in angular KE = loss in PE

If l = length of the pole, moment of inertial of the pole about the edge $= M\left[\frac{l^2}{12} + \frac{l^2}{4}\right] = \frac{Ml^2}{3}$

Loss in potential energy $= \frac{Mgl}{2}$

Gain in angular $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{Ml^2}{3} \times \omega^2$

$\therefore$ $\frac{1}{2}\frac{Ml}{3}\omega^2 = \frac{Mgl}{2}$ or $(l\omega)^2 = 3gl$

or $l\omega = v = \sqrt{3gl}$

$= \sqrt{3 \times 10 \times 30} = 30 ms^{-1}$

14. Let the velocity of the scooter be v ms^{-1}. Then (v – 10)100 = 100 or v = 20 ms^{-1}

15. Let x be the distance between the particles after t second. Then

$$x = vt - \frac{1}{2}at^2 \qquad .....(i)$$

For x to be maximum,

$$\frac{dx}{dt} = 0$$

or v – at = 0

or $t = \frac{v}{a}$

Putting this value in eqn. (i), we get;

$$x = v\left(\frac{v}{a}\right) - \frac{1}{2}a\left(\frac{v}{a}\right)^2 = \frac{v^2}{2a}$$

16. $\frac{dv}{dt} = a$

or $\int_{v_1}^{v_2} dv = \int a\, dt$

$\therefore$ Δv = Area under a – t graph,

where, Δv = magnitude of change in velocity.

17. $-s = ut_1 - \frac{1}{2}gt_1^2 \qquad ......(i)$

$$-s = -ut_3 - \frac{1}{2}gt_3^2 \qquad .....(ii)$$

$$-s = -\frac{1}{2}gt_2^2 \qquad .....(iii)$$

$-st_3 = ut_1t_3 - \frac{1}{2}gt_1^2t_3$ ....(iv)

$-st_1 = -ut_1t_3 - \frac{1}{2}gt_3^2t_1$ ...... (v)

Adding, $-s(t_1 + t_3) = -\frac{1}{2}gt_3t_1(t_3 + t_1)$ .... (v)

Adding, $-s(t_1 + t_3) = -\frac{1}{2}gt_3t_1(t_3 + t_1)$

or $s = +\frac{1}{2}gt_2t_1$ ...... (vi)

From equns. (iii) and (vi),

$\frac{1}{2}gt_3t_1 = \frac{1}{2}gt_2^2$

$\therefore$ $t_2 = \sqrt{t_3t_1}$

18. $u = 0$, $a = 2\ m/s^2$, $t = 10$ sec

$\therefore$ $s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 100$

$= 100$ m

Velocity after 10 sec,

$v = u + at = 0 + 2 \times 10 = 20$ m/s

$\therefore$ $s_2 = v \times 30 = 20 \times 30 = 600$ m

Final velocity = 0, $a = -4\ m/s^2$

$\therefore$ $0 = v^2 + 2as_3$

$0 = (20)^2 - 2 \times 4 \times s_3$

$\therefore$ $s_3 = \frac{400}{8} = 50$ m

19. Displacement in horizontal direction = $\pi R = \pi m$. Displacement in vertical direction = $2R = 2$ m.

$\therefore$ Resultant displacement $= \sqrt{\pi^2 + 4}$ m

20. $\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m} = \left(\frac{6t^2\hat{i} + 4t\hat{j}}{3}\right) m/s^2$

$\therefore$ $\vec{v} = \int_0^3 \left(\frac{6t^2}{3}\hat{i} + \frac{4t}{3}\hat{j}\right)dt$

$= \left[\frac{6t^3}{9}\hat{i} + \frac{4t^2}{6} - \hat{j}\right]_0^3 = 18\hat{i} + 6\hat{j}$

21. We know that the speed of an object, falling freely under gravity, depends only upon its height from which it is allowed to fall and not upon its mass. Since, the paths are frictionless and all the objects are falling through the same vertical height, therefore their speeds on reaching the ground must be same or ratio of their speeds = 1 : 1 : 1

22. $x = \alpha t^3 + \beta t^2 + \gamma t + \delta$

$v = \text{velocity} = \frac{dx}{dt}$

$3\alpha t^2 + 2\beta t + \gamma$

v_0 = Initial velocity (at t = 0) = γ

Similarly, acceleration

$a = \frac{dv}{dt} = 6\alpha t = 2\beta$

Initial acceleration when t = 0

$a_0 = 2\beta$

$\therefore \frac{a_0}{v_0} = \frac{2\beta}{\gamma}$

i.e., $\frac{a_0}{v_0} \propto \frac{\beta}{\gamma}$

23. Velocity of the thief's car with respect to ground is, v_{TG} = 10 m/s

Velocity of police man with respect to ground = v_{PG} = 5 m/s

Velocity of bullet fired by police man with respect to ground,

$v_{BP} = 72 km/h = \frac{72 \times 5}{18} = 20\ m/s$

Velocity with which bullet will hit the target is,

$v_{BT} = v_{BG} + v_{GT}$

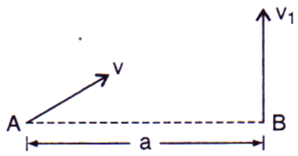
$= v_{BP} + v_{PG} + v_{GT}$

= 20 + 5 – 10 = 15 m/s.

24. $t = \frac{a}{u'}$

$= \frac{a}{\sqrt{v^2 - v_1^2}}$

$= \sqrt{\frac{a^2}{v^2 - v_1^2}}$



25 As $v^2 = u^2 + 2as$

$\therefore$ $u^2 \propto s$ .....(i)

For given condition:

$u'^2 \propto 3s$ ....(ii)

From equations (i) and (ii),

$\frac{u'^2}{u^2} = 3$ or $u' = \sqrt{3}\ v_0$ $(\because u = v_0)$

26. $x = 40 + 12t - t^3$

$\therefore$ Velocity, $v = \frac{dx}{dt} = 12 - 3t^2$

When particle comes to rest,

$\frac{dx}{dt} = v = 0$

$\therefore$ $13 - 3t^0 = 0$

or $3t^2 = 13$ or t = 2 sec

Distance travelled by the particle before coming to rest:

$\int_0^s ds = \int_0^2 v dt$

$\therefore \quad S = \int_0^2 (12 - 3t^2)dt = \left[12t - \frac{3t^3}{3}\right]_0^2$

= 12 × 2 – 8 = 16 m

27. Time taken by a body to fall a height h to reach the ground is,

$t = \sqrt{\frac{2h}{g}}$

$\therefore \quad \frac{t_A}{t_B} = \frac{\sqrt{2h_A / g}}{\sqrt{2h_B / g}} = \sqrt{\frac{h_A}{h_B}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

28. The velocity upstream is (3 – 2) km/hr and down the stream is (3 + 2)km/hr.

$\therefore \quad \text{Total time taken} = \frac{2\text{ km}}{1\text{ km/hr}} + \frac{2\text{km}}{5\text{km/hr}} = 2.4\text{ hrs}$

29. $v^2 - u^2 = 2as$ or $6^2 - u^2 = 2a \times 5$
and $8^2 - u^2 = 2a(5 + 7) = 2a \times 12$
solving, $a = 2\text{ m/s}^2$ and $u = 4$ m/s

30. $a = \frac{dv}{dt} = 6t + 5$

or $dv = (6t + 5)dt$
Integrating it, we have;

$\int_0^v dv = \int_0^t (6t + 5)dt$

$\therefore \quad v = \frac{6t^2}{2} + 5t + C$

(where C is constant of integration)
where t = 0, v = 0, so C = 0

$\therefore \quad v = \frac{ds}{dt} = 3t^2 + 5t$

or $ds = (3t^2 + 5t)dt$
Integrating it within conditions of motion, i.e., as t changes from 0 to 2s, s changes from 0 to s, we have;

$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$

$\therefore \quad \int_0^s ds = \int_0^2 (3t^2 + 5t)dt$

[CHEMISTRY]

31. (b) Complete $E.C. = [Ar]^{18} 3d^{10} 4s^2 4p^6$.
32. (a) In case of N^{3-}, $p = 7$ and $c = 10$
33. (b) According to the Bohr model atoms or ions contain one electron.
34. (b) Both He and Li^+ contain 2 electrons each.
35.

36. For He^+ ion, $\frac{1}{\lambda} = Z^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$(2)^2 R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{4}$

For hydrogen atom, $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{3R}{4} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ or $\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{3}{4}$

$n_1 = 1$ and $n_2 = 2$

37.

Species	$_{19}K^+$	$_{20}Ca^{2+}$	$_{21}Sc^{3+}$	$_{17}Cl^-$
No. of electrons	18	18	18	18

38. For H like particles, $E_n = \frac{-21.78 \times 10^{-19}}{n^2} Z^2$

$= \frac{-21.78 \times 10^{-19}}{1^2} \times (2)^2 = -8.712 \times 10^{-18}$ J

39. Ionisation potential of hydrogen = 13.6 eV

$\therefore$ $E_1 = -13.6$ eV

and $E_n = -\frac{E_1}{n^2} = \frac{-13.6}{4} = -3.4$eV

Hence, 3.4 eV energy is required to remove an electron from n = 2 state of hydrogen atom.

40. Ionisation enthalpy $\propto Z^2$ (where Z = Atomic number)

I.E. of Li^{2+} = I.E. of $He^+ \times \left(\frac{3^2}{2^2}\right)$

I.E. of $Li^{2+} = 19.6 \times 10^{-18} \times \frac{9}{4} = 44.1 \times 10^{-18}$ J atom^{-1}

41. $E_n = E_1 \times \frac{Z^2}{n^2}$

E_1 =energy of hydrogen in first orbit, n = 2, Z = atomic number

$E = -13.6 \times \frac{(3)^2}{(2)^2} = -30.6$ eV

42. $I.E._{(He^+)} = 19.6 \times 10^{-18}$ J atom^{-1}

$I.E._{He^+} = E_\infty - E_1(\text{for } He^+) = 0 - E_1(\text{for } He^+)$

$\therefore$ E_1 (for He^+) = – 19.6 × 10^{-18} J atom^{-1}

$\frac{-KZ^2}{n^2}$ J atom$^{-1} = -19.6 \times 10^{-18}$ J atom^{-1} $(\because Z = 2, n = 1)$

4K = 19.6 × 10^{-18} J atom^{-1} ⇒ K = 4.9 × 10^{-18} J atom^{-1}

$E_1(\text{for } Li^{2+}) = \frac{-KZ^2}{n^2} = -4.9 \times 10^{-18} \times \frac{3^2}{1^2}$ $(\because Z = 3, n = 1)$

= – 4.41 × 10^{-17} J atom^{-1}

43. E_H in first orbit $= \frac{-19.6\times10^{-18}}{4}$ J

$E_{Be^{3+}}$ in second orbit

$= -\left(\frac{19.6\times10^{-18}}{4}\right)\times\frac{16}{4}$

$= -16.6 \times 10^{-18}$ J

44. Angular momentum, $mvr = n\frac{h}{2\pi}$ (n = 2 for first excited state)

45. -

46. Be^{3+} is hydrogenic ions, i.e., consits of one extranuclear electron.

47. Orbital angular momentum $= \sqrt{\ell(\ell+1)}\frac{h}{2\pi}; \ell = 1$ for p-orbital

48. $\frac{1}{\lambda} = \bar{v} = R_H Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right); \frac{1}{\lambda_{He^+}} = \bar{v}_{He^+} = R_H \times 2^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$

$= R_H\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{1}{\lambda_H}$ for $n = 2$ to $n_1 = 1$

49. -

50. Number of angular nodes $\ell = 2$($\ell = 2$ for d-orbital) Orbital angular momentum $= \sqrt{\ell(\ell+1)}\frac{h}{2\pi} = \sqrt{2(2+1)}\frac{h}{2\pi}$

51. Use (n + l) rule.

52. For g-subhell, l = 4. Hence, n = 5 (minimum)

Number of orbitals in 5th shell = $n^2 = 5^2 = 25$

53. -

54. 7s orbital, (n + l) is minimum with n minimum.

55. Number of radial nodes = n – l – 1

56. Z for He^+ = 2, first excited state for He^+, n= 2

Z for Li^2 = 3; second excited state, n = 3

$E_n = -13.6\ Z^2/n^2$

57. Cr^{3+} – $[Ar]3d^3$, U.P. = 3; Fe^{2+} – $[Ar]3d^6$,

Unpaired electrons = 4; Ni^{2+} – $[Ar]3d^8$,

Unpaired electrons = 2

Cu^{2+} – $[Ar]3d^9$, Unpaired electrons = 1

58. -

59. For 3p orbital n = 3, l = 1

No.f spherical nodes = n – l – 1 = 3 – 1 – 1 = 1

No. of non-spherical nodes = l = 1

60. -

[MATHEMATICS]

61. (a) $x=\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$ to ∞

We have $x=\sqrt{1+x}$

$\Rightarrow x^2=1+x \Rightarrow x^2-x-1=0$

$\Rightarrow x=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}$

As $x>0$, we get $x=\frac{1+\sqrt{5}}{2}$

62. (c) When $x<0$, $|x|=-x$

$\therefore$ Equation is $x^2-x-6=0 \Rightarrow x=-2,3$

$\because x<0, \therefore x=-2$ is the solution.

When $x\ge 0$, $|x|=x$

$\therefore$ Equation is $x^2+x-6=0 \Rightarrow x=2,-3$

$\because x\ge 0$, $\therefore x=2$ is the solution.

Hence $x=2$, -2 are the solutions and their sum is zero.

Aliter : $|x^2|+|x|-6=0$

$\Rightarrow (|x|+3)(|x|-2)=0$

$\Rightarrow |x|=-3$, which is not possible and $|x|=2$

$\Rightarrow x=\pm 2$.

63. (c) Given equations are $2x^2+3x+5\lambda=0$ and $x^2+2x+3\lambda=0$ have a common root if

$\frac{x^2}{-\lambda}=\frac{x}{-\lambda}=\frac{1}{1} \Rightarrow x^2=-\lambda, x=-\lambda$ or $\lambda=-1, 0$.

64. (a) We have equal roots, therefore $\lambda^2=4\mu$.

Now second equation $x^2+\lambda x-12=0$ has a root 2, so put $x=2 \Rightarrow 4+2\lambda-12=0 \Rightarrow \lambda=4$

Hence from $\lambda^2=4\mu$, we have $\mu=\frac{16}{4}=4$

$\Rightarrow (\lambda,\mu)=(4,4)$.

65. (a) From $k=\frac{x^2-x+1}{x^2+x+1}$

We have $x^2(k-1)+x(k+1)+k-1=0$

As given, x is real $\Rightarrow (k+1)^2-4(k-1)^2\ge 0$

$\Rightarrow 3k^2-10k+3\ge 0$

Which is possible only when the value of k lies between the roots of the equation $3k^2-10k+3=0$

That is, when $\frac{1}{3}\le k\le 3$ {Since roots are $\frac{1}{3}$ and 3}

66. (d) We know that the expression $ax^2+bx+c>0$ for all x, if $a>0$ and $b^2<4ac$

$\therefore (a^2-1)x^2+2(a-1)x+2$ is positive for all x if

$a^2-1>0$ and $4(a-1)^2-8(a^2-1)<0$

$\Rightarrow a^2-1>0$ and $-4(a-1)(a+3)<0$

$\Rightarrow a^2-1>0$ and $(a-1)(a+3)>0$

$\Rightarrow a^2>1$ and $a<-3$ or $a>1$ $\quad \Rightarrow a<-3$ or $a>1$

67. (a) Given equation can be written as

$(m+1)x^2-\{m(a+b)-(a-b)\}x+c(m-1)=0$.

Roots are equal and of opposite sign. So sum of roots is equal to zero.

$\Rightarrow 0=m(a+b)-(a-b) \Rightarrow m=\frac{a-b}{a+b}$.

68. (b) Let the equation (in correctly written form) be $x^2+17x+q=0$. Roots are -2, -15. So $30=q$, so correct equation is $x^2+13x+30=0$. Hence roots are -3, -10.

69. (b) Let the roots are α and $n\alpha$

Sum of roots, $\alpha+n\alpha=-\frac{b}{a} \Rightarrow \alpha=-\frac{b}{a(n+1)}$ .....(i)

and product, $\alpha.n\alpha=\frac{c}{a} \Rightarrow \alpha^2=\frac{c}{na}$ ....(ii)

From (i) and (ii), we get

$\Rightarrow \left[-\frac{b}{a(n+1)}\right]^2=\frac{c}{na} \Rightarrow \frac{b^2}{a^2(n+1)^2}=\frac{c}{na}$

$\Rightarrow nb^2=ac(n+1)^2$.

Note : Students should remember this question as a fact.

70. (a) As given, $\sin\alpha+\cos\alpha=-\frac{b}{a}$, $\sin\alpha\cos\alpha=\frac{c}{a}$

To eliminate α, we have

$1=\sin^2\alpha+\cos^2\alpha=(\sin\alpha+\cos\alpha)^2-2\sin\alpha\cos\alpha$

$=\frac{b^2}{a^2}-\frac{2c}{a} \Rightarrow a^2-b^2+2ac=0$

71. (a) $x^2-2kx+k^2+k-5=0$

Roots are less than 5, $D\ge 0$

$4k^2-4(k^2+k-5)\ge 0$ ……(i)

$\Rightarrow k\le 5 \Rightarrow f(5)>0$ .....(ii)

$\Rightarrow k\in(-\infty,4)\cup(5,\infty)$; $-\left(\frac{2k}{2}\right)<5 \Rightarrow k<5$ …..(iii)

form (i), (ii) and (iii), $k\in(-\infty,4)$

72. (d) Let the roots are α, β of $x^2 - bx + c = 0$ and α', β' be roots of $x^2 - cx + b = 0$

Now $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{b^2 - 4c}$ .....(i)

and $\alpha' - \beta' = \sqrt{(\alpha'+\beta')^2 - 4\alpha'\beta'} = \sqrt{c^2 - 4b}$ .....(ii)

But $\alpha - \beta = \alpha' - \beta'$

$\Rightarrow \sqrt{b^2 - 4c} = \sqrt{c^2 - 4b} \Rightarrow b^2 - 4c = c^2 - 4b$

$\Rightarrow b^2 - c^2 = 4c - 4b$

$\Rightarrow (b+c)(b-c) = 4(c-b) \Rightarrow b + c = -4$

73. (b) Given equation can be written as

$x^2 - 3kx + 2k^2 - 1 = 0$

So the product of roots is $2k^2 - 1$. But the product of roots is 7. Hence $2k^2 - 1 = 7 \Rightarrow 2k^2 = 8 \Rightarrow k = \pm 2$

But k cannot be negative.

74. (b) Given that $5\cos A + 3 = 0$ or $\cos A = -\frac{3}{5}$

Let $\alpha = \sin A$ and $\beta = \tan A$, then the sum of roots $= \alpha + \beta = \sin A + \tan A$

$= \sin A + \frac{\sin A}{\cos A} = \frac{\sin A}{\cos A}(1 + \cos A)$

$= \frac{\sqrt{1 - 9/25}}{-3/5}\left(1 - \frac{3}{5}\right) = \frac{4}{-5}.\frac{5}{3}.\frac{2}{5} = \frac{8}{-15}$

and product of roots $\alpha.\beta = \sin A \tan A = \frac{\sin^2 A}{\cos A}$

$= \frac{16/25}{-3/5} = -\frac{16}{25} \times \frac{5}{3} = -\frac{16}{15}$

Thus required equation is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow \quad x^2 + \frac{8x}{15} - \frac{16}{15} = 0 \Rightarrow 15x^2 + 8x - 16 = 0$

75. (b) If one root is square of other of the equation $ax^2 + bx + c = 0$, then $b^3 + ac^2 + a^2c = 3abc$

Which can be written in the form $a(c-b)^3 = c(a-b)^3$

Trick : Let roots be 2 and 4, then the equation is $x^2 - 6x + 8 = 0$. Here obviously

$X = \frac{a(c-b)^3}{c} = \frac{1(14)^3}{8} = \frac{14}{2} \times \frac{14}{2} \times \frac{14}{2} = 7^3$

Which is given by $(a-b)^3 = 7^3$.

76. (d) 8, 2 are the roots of $x^2 + ax + \beta = 0$

$\therefore\ 8 + 2 = 10 = -a, 8.2 = 16 = \beta$ i.e. $a = -10, \beta = 16$

3, 3 are the roots of $x^2 + \alpha x + b = 0$

$\therefore\ 3 + 3 = 6 = -\alpha,\ 3.3 = b$ i.e. $\alpha = -6, b = 9$

Now, $x^2 + ax + b = 0$ becomes $x^2 - 10x + 9 = 0$

or $(x-1)(x-9) = 0 \Rightarrow x = 1, 9$.

77. (b) **Trick** : By inspection, we see that all the values of x lying in $(-\infty, 1) \cup (2, 3)$ satisfy the equations and no other value outside the interval satisfy it.

78. (c) We have $\alpha + \beta = -\frac{7}{2}$ and $\alpha\beta = \frac{c}{2}$

$\therefore\ |\alpha^2 - \beta^2| = \frac{7}{4} \Rightarrow \alpha^2 - \beta^2 = \pm\frac{7}{4}$

$\Rightarrow (\alpha+\beta)(\alpha-\beta) = \pm\frac{7}{4} \Rightarrow -\frac{7}{2}\sqrt{\frac{49}{4} - 2c} = \pm\frac{7}{4}$

$\Rightarrow \sqrt{49 - 8c} = \mp 1 \Rightarrow 49 - 8c = 1 \Rightarrow c = 6$

79. (c) Given equation is $x^2 + (2+\lambda)x - \frac{1}{2}(1+\lambda) = 0$

So, $\alpha + \beta = -(2+\lambda) = 0$ and $\alpha\beta = -\left(\frac{1+\lambda}{2}\right)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = [-(2+\lambda)]^2 + 2.\frac{(1+\lambda)}{2}$

$\Rightarrow \alpha^2 + \beta^2 = \lambda^2 + 4 + 4\lambda + 1 + \lambda = \lambda^2 + 5\lambda + 5$

which is minimum for $\lambda = 1/2$.

80. (a) Given equation $x^2 - |x| - 6 = 0$

If $x > 0$, $\therefore$ equation is $x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0 \Rightarrow x = 3, x = -2 \Rightarrow x = 3$

If $x < 0$, $\therefore$ equation is $x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) = 0 \Rightarrow x = -3, x = 2 \Rightarrow x = -3$

Hence product of all possible real roots = – 9.

81. (c) $\alpha + \alpha^2 = -\frac{p}{3}$ and $\alpha.\alpha^2 = 1$.

So $\alpha = 1, \omega, \omega^2$. If $\alpha = 1, p < 0$.

If $\alpha = \omega$ or ω^2, we have $\omega + \omega^2 = -\frac{p}{3}$

$\Rightarrow\ -1 = \frac{-p}{3}\ \Rightarrow p = 3$.

82. (d) $a + b = 3$, $ab = 1$ and $-p = a - 2 + b - 2$, $q = (a-2)(b-2)$

$\Rightarrow -p = a + b - 4$, $q = ab - 2(a+b) + 4$

$\Rightarrow -p = 3 - 4$ and $q = 1 - 2(3) + 4 \Rightarrow (p, q) = (1, -1)$.

83. (a) Let the roots are α and 2α

$\Rightarrow \alpha + 2\alpha = \frac{1-3a}{a^2 - 5a + 3}$ and $\alpha.2\alpha = \frac{2}{a^2 - 5a + 3}$

$\Rightarrow 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2 - 5a + 3)^2}\right] = \frac{2}{a^2 - 5a + 3}$

$\Rightarrow \frac{(1-3a)^2}{(a^2 - 5a + 3)} = 9 \Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$

$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$.

84. (d) Given equations are $x^2 - 3x + a = 0$ ……(i)

and $x^2 + ax - 3 = 0$ ……(ii)

Subtracting (ii) from (i), we get

$\Rightarrow -3x - ax + a + 3 = 0$

$\Rightarrow (a+3)(-x+1) = 0$

$\Rightarrow$ either $a = -3$ or $x = 1$

When $a = -3$, the two equations are identical. So, we take $x = 1$, which is the common root of the two equations. Substituting $x = 1$ in (i), we get $1 + a - 3 = 0 \Rightarrow a = 2$.

85. (a) If $(x+1)$ is a factor of

$x^4 - (p-3)x^3 - (3p-5)x^2 + (2p-7)x + 6$, then by putting $x = -1$, we get

$1 + (p-3) - (3p-5) - (2p-7) + 6 = 0$

$\Rightarrow -4p = -16 \Rightarrow p = 4$.

86. (d) The given condition suggest that a lies between the roots. Let $f(x) = 2x^2 - 2(2a+1)x + a(a+1)$

For 'a' to lie between the roots we must have Discriminant ≥ 0 and $f(a) < 0$.

Now, Discriminant ≥ 0

$\Rightarrow 4(2a+1)^2 - 8a(a+1) \geq 0$

$\Rightarrow 8(a^2 + a + 1/2) \geq 0$ which is always true.

Also $f(a) < 0 \Rightarrow 2a^2 - 2a(2a+1) + a(a+1) < 0$

$\Rightarrow -a^2 - a < 0 \Rightarrow a^2 + a > 0 \Rightarrow a(1+a) > 0$

$\Rightarrow a > 0$ or $a < -1$.

87. (b) α, β, γ are the roots of the equation

$x^3 + ax^2 + bx + c = 0$

so $\alpha + \beta + \gamma = -a$, $\alpha\beta + \beta\gamma + \gamma\alpha = b$ and $\alpha\beta\gamma = -c$

Now $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$

$= -b/c$.

88. (c) Given $\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1}$

$\Rightarrow \frac{2x}{(2x+1)(x+2)} > \frac{1}{(x+1)}$

$\Rightarrow \frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$\Rightarrow \frac{2x(x+1) - (2x+1)(x+2)}{(x+1)(2x+1)(x+2)} > 0$

$\Rightarrow \frac{2x^2 + 2x - 2x^2 - 4x - x - 2}{(x+1)(x+2)(2x+1)} > 0$

$\Rightarrow \frac{-3x - 2}{(x+1)(x+2)(2x+1)} > 0$

Equating each factor equal to 0, we have

$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$.

It is clear that $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$.

89. (a) $ax^2 - 2x + 4 > 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4 - 16a}}{2a} \Rightarrow x = \frac{1 \pm \sqrt{1-4a}}{a}$

$\therefore \frac{1 - \sqrt{1-4a}}{a} < x < \frac{1 + \sqrt{1-4a}}{a}$.

90. (a) Let required roots are $3\alpha, 2\alpha, \beta$

($\because$ ratio of two roots are $3:2$)

$\therefore \sum\alpha = 3\alpha + 2\alpha + \beta = \frac{-(-9)}{1} = 9$

$\Rightarrow 5\alpha + \beta = 9$ …..(i)

$\sum\alpha\beta = 3\alpha.2\alpha + 2\alpha.\beta + \beta.3\alpha = 14$

$\Rightarrow 5\alpha\beta + 6\alpha^2 = 14$ …..(ii)

and $\sum\alpha\beta\gamma = 3\alpha.2\alpha.\beta = -24$

$\Rightarrow 6\alpha^2\beta = -24$ or $\alpha^2\beta = -4$ …..(iii)

from (i), $\beta = 9 - 5\alpha$, put the value of β in (ii)

$\Rightarrow 5\alpha(9 - 5\alpha) + 6\alpha^2 = 14$

$\Rightarrow 19\alpha^2 - 45\alpha + 14 = 0$

$\Rightarrow (\alpha - 2)(19\alpha - 7) = 0$

$\therefore \alpha = 2$ or $\frac{7}{19}$

$\therefore$ from (i) , if $\alpha = 2$, then $\beta = 9 - 5 \times 2 = -1$

$\because \alpha = 2, \beta = -1$ satisfy the equation (iii) so required roots are 6, 4, –1.